PREDICTING DURATION SPECIFIC AVERAGES AND VARIANCES OF LIVE BIRTHS: APPLICATION OF A STOCHASTIC MODEL OF HUMAN REPRODUCTION

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Introduction

To measure the fertility level in a society one can use many available measures, such as the General Fertility Rate, age specific fertility rates or duration specific birth rates. The use of a particular measure depends on the availability of data and the purpose of the study. In the absence of the required data, one often turns towards some suitable model specific to the purpose in mind. At times these models are used without adequately testing them with some observed set of data obtained from field surveys or from other sources. In this paper we shall first predict duration specific averages and variances of live births by using the Perrin and Sheps[1] stochastic model of human reproduction and then shall compare them with the corresponding observed values obtained directly from a sample of nonwestern women.

In the last few decades a number of hypothetical models for predicting fertility levels have been developed by Dandekar[2], Brass[3], Potter[4], Henry[5], and Perrin and Sheps[6]. In this paper we will confine our discussions to the stochastic model of human reproduction developed by Perrin and Sheps. This model has been applied by other investigators to study a variety of problems. For example, Sheps and Perrin[7,8] used it to study the effects of contraception on birth rates and to study the distributions of birth intervals; Sheps[9] used it to study the effect of pregnancy wastage on fertility, and Potter et. al.[10] used it to study birth interval dynamics. Sheps and Perrin have mainly used hypothetical values of the basic parameters of the model, whereas Potter et. al. used data for Indian women from the Khanna study in which the users were not separated from the nonusers of contraception. The Perrin and Sheps model, however, is properly applicable only to nonusers of contraception and we have an opportunity to compare some of the theoretical results obtained from this model with the corresponding observed values for nonusers. These comparisons do not neccessarily provide a conclusive test of the model mainly because the magnitude of divergence between the observed and theoretical values reflects not only the limitations of the model but also dificiencies in the data.

Data

The data are taken from the results of an intensive fertility survey conducted in Taichung City of Taiwan, by the Taiwan Population Studies Center, in collaboration with the Population Studies Center of the University of Michigan. The Taichung Survey[11] is based on a probability sample of 2,443 married women between the ages 20 to 39, living with their husbands, interviewed towards the end of 1962 just before a year long family planning action program.[12] Extensive information about the various demographic and socio-economic characteristics, attitudes towards family planning and practice of various family planning methods are available for each woman. In this analysis, however, only a fraction of this information is used.

The Taichung sample provides detailed information about the pregnancy histories of all women in the sample. In this sample about 84 per cent of the couples in the childbearing ages had never used contraception (excluding induced abortion) between marriage and the last pregnancy prior to interview. Even for the remaining 16 per cent who had used contraception at some time, we can use that part of their pregnancy histories in which contraception was not used. Thus, the Taichung survey provided a set of pregnancy histories free from contraception which can be used to estimate the parameters in the absence of contraception, required in the Perrin and Sheps model.

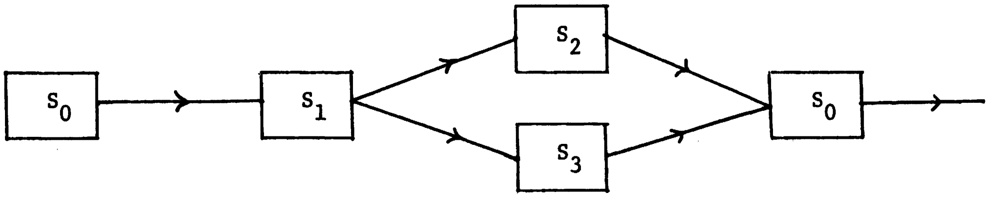
Before presenting the specific comparisons let us first review some of the basic assumptions underlying the Perrin and Sheps model and consider how well these assumptions are met by our data.

The Model[13]

The Perrin and Sheps model assumes that in a given month a woman can only be found in one of the four mutually-exclusive states: a nonpregnant, fecundable state S_0; a pregnant state S_1; an infecundable period following a pregnancy loss in state S_2; and an infecundable period following a live birth in state S_3. Here pregnancy loss includes spontaneous abortions, induced abortions, and still births, whereas in the orginal version still births were separated from spontaneous and induced abortions. The model further assumes that at the time of marriage women start in the nonpregnant fecundable state S_0, during which they are subject to some fixed probability of conception. After a random length of time they pass to state S_1. The duration of their stay in state S_1 depends upon the outcome of the pregnancy. Following the termination of a pregnancy, women pass to state S_2 or state S_3. If the pregnancy terminates in a pregnancy loss, women pass from state S_1 to state S_2; and if the pregnancy terminates in a live birth women pass to state S_3. During their stay in states S_2 or S_3 women are temporarily infecundable, i.e., the probability of conception during their stay in these states is zero. From states S_2 or S_3 women pass to state S_0 and become fecundable again, and thus the process starts once more.

In the "pure" version of the model, women are assumed to be homogeneous with respect to its parameters, namely, the moments of the

Schematically:



(The length of stay in each state, and the outcome of each pregnancy are independent random variables.)

S_0- Nonpregnant fecundable state

S_1- Pregnant State

S_2- Infecundable period following pregnancy losses

S_3- Infecundable period following live births

length of stay in states S_0, S_1, S_2, and S_3, and the probabilities of moving from state S_1 to state S_2 or state S_3. These parameters are assumed to be independent of women's age and the pregnancy order. The length of stay in each state and the outcome of pregnancy are assumed to be independent random variables. Moreover, the length of the reproductive period is assumed to be unlimited. In actuality as had been shown elsewhere[14], Taiwanese women in the present sample are not homogeneous, the parameters are dependent on women's age and the pregnancy order, and the reproductive period is limited. Since we are using the values of parameters estimated from real data in this "modified" version of the model, we are roughly taking into account the heterogeneity among women with respect to their age and the pregnancy order. For example, we estimated the average and variance of conception delay and of pregnancy intervals by using actual frequency distributions of all heterogeneous women observed between marriage and interview, and similarly we estimated the probability of pregnancy losses from all pregnancies during this period.[15] For applying the results of the Perrin and Sheps model, we assume that all women included in the analysis are homogeneous with respect to these estimated overall parameters, and that these overall parameters remain constant throughout the observed reproductive period.

Two assumptions, homogeneity and stability of the parameters, are only partly taken care of in this modified version of the model. The other two assumptions of the model, independence between different states and an unlimited reproductive period, are not met at all in actuality. Considering the limitations of the model, Perrin and Sheps suggested that "the period for which this model can be assumed to hold for each woman must necessarily be restricted to an interval of at most 10 to 15 years in the middle of the childbearing age."[16] Given these limitations of the model and the data, we do not expect that the observed averages and variances will agree closely with the corresponding estimated values. Nevertheless, it is useful to measure how small or great are the discrepancies between the values observed in a real situation and the values predicted by a model which has the virtue of simplicity.

Results

The Perrin and Sheps model provides approximate mathematical expressions for estimating averages and variances of the number of live births at the end of "t" months after marriage. These are shown in the appendix along with the corresponding estimation procedures.

As noted by Perrin and Sheps[17] these expressions yield good approximations to exact moments after five years of marriage, i.e., after five years of marriage the two moments of live births obtained by using approximate expressions are the same as those obtained by using exact expressions. For using these approximate expressions one requires the estimates of the first two moments of the first passage time from state S_0 to state S_3 or the first birth interval (FBI); and the first three moments of the time between the two successive visits to state S_3 or completed birth interval (BI). One can estimate these moments of FBI and BI either from the sample frequency distributions of FBI and BI or by using the results of the Perrin and Sheps model. For the latter case one needs to estimate the basic parameters of the model from the observed data. In this paper we have estimated the moments of FBI and BI by using both of these procedures. These results are compared in the appendix (Table 3).

Substituting the estimated moments of FBI and BI from the first set in expressions for the average and the variance of live births per woman after t months of marriage, we obtained two expressions: $0.0375\ t - 0.1233$ for average live births and $0.00602\ t + 0.24210$ for variance of live births. By substituting $t = 1, 2, 3, \ldots$ etc., we can obtain the theoretical number of live births and also their variances in one, two,... etc. months after marriage. These theoretical results will be referred as the 'Theoretical A' series.

Similarly by substituting the estimated moments of FBI and BI from the second set we obtained two expressions: $0.03590\ t - 0.0888$ for average live births and $0.00654\ t + 0.13590$ for variance of live births. The results from these expressions will be referred as the 'Theoretical B' series.

Table 1. Comparison of Observed and Theoretical Averages of Live Births per Woman in T Years of Marriage in the Absence of Contraception.

Marriage duration (years)	Observed[a/]	Theoretical[b/] A	Theoretical[b/] B	I[c/] A	I[c/] B
1	.302	.327	.342	8.3	13.2
2	.823	.777	.773	-5.6	-6.1
3	1.222	1.227	1.204	.4	-1.5
4	1.674	1.667	1.634	.2	-2.4
5	2.042	2.127	2.065	4.2	1.1
6	2.449	2.577	2.496	5.2	1.9
7	2.822	3.027	2.927	7.3	3.7
8	3.152	3.477	3.358	10.3	6.5
9	3.479	3.927	3.788	12.9	8.9
10	3.800	4.377	4.219	15.2	11.0
11	4.130	4.827	4.650	16.9	12.6
12	4.351	5.277	5.081	21.3	16.8
13	4.566	5.727	5.512	25.4	20.7
14	4.784	6.177	5.942	29.1	24.2
15	5.022	6.627	6.373	32.0	26.9
16	5.105	7.077	6.804	38.6	33.3
17	5.202	7.527	7.235	44.7	39.1
18	5.194	7.977	7.666	53.6	47.6
19	5.347	8.427	8.096	57.6	51.4
20	5.303	8.877	8.527	67.4	60.8

a/ Women who were premaritally pregnant or who have used contraception between marriage and last pregnancy are excluded.

b/ Theoretical average live births per woman in T years of marriage:

$$A = 0.4500\ T - 0.1233$$

$$B = 0.4308\ T - 0.0888$$

c/ $I = \frac{\text{Theoretical - Observed}}{\text{Observed}} \times 100$

We compare the above two series with the averages and variances of live births after t months of marriage obtained from the sample. If we consider women married for at least t months and their live births during these months, we frequency distributions of women by number of live births for each month after marriage. From these observed frequency distributions the averages and variances of live births can be calculated for each successive month following marriage. However, for simplicity the moments of live births are calculated only for one year intervals, i.e., for women married for at least 12, 24, 36, 48,..., etc. months. The observed and estimated values of average live births are compared in Table 1 and in Figure 1, and the corresponding values for variance of live births are compared in Table 2 and Figure 2.

The differences between the two theoretical series A and B are very small. For this reason we discuss in the following paragraphs only the differences between theoretical series B and the observed values. The differences between the observed and the predicted averages and variances of live births during the first five years of marriage could be attributed to the fact that here we used approximate expressions instead of exact expressions for predicting these averages and variances. Hence, we shall restrict our discussion to the differences after the fifth year of marriage.

As can be seen from the last column in Table 1, the differences between the observed and the predicted number of live births increase gradually with the duration of marriage. For example, for women married six years or more, the theoretical average number of live births during six years of marriage is only two percent (or about .05 births) higher than the corresponding observed average number of live births. But this difference increases gradually to 61 per cent (or about three births) for women married 20 years or more. (See Table 1 and Figure 1).

Comparisons of variances in Table 2 and Figure 2 show a different pattern. Except for the second year, the theoretical variances are always lower than the observed. For example, for women married at least six years the theoretical variance is 20 per cent (or 0.15) lower than the observed variance. This difference increases gradually to 53 per cent (or 1.9) for women married for at least 20 years. (See Table 2 and Figure 2)

These patterns in the differences could arise due to variety of reasons. Three reasons are discussed here.

Figure 1: Theoretical and Observed Number of Live Births by Duration of Marriage

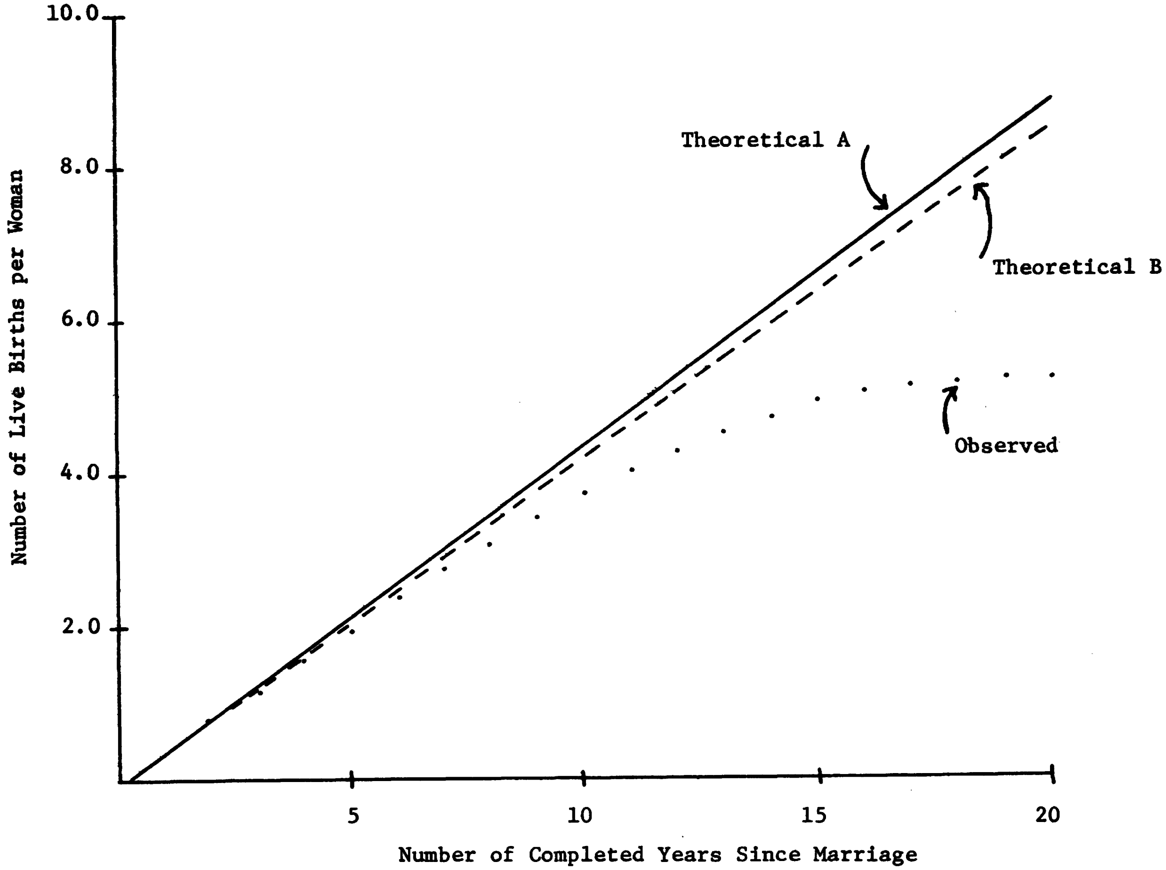


Table 2. Comparison of Observed and Theoretical Variance of Live Births per Woman in T Years of Marriage in the Absence of Contraception.

Marriage duration (years)	Observed[a]	Theoretical[b]		I[c]	
		A	B	A	B
1	.215	.314	.214	46.0	-.5
2	.205	.387	.293	88.8	42.9
3	.382	.459	.371	20.2	-2.9
4	.478	.531	.450	11.1	-5.9
5	.630	.603	.528	-4.3	-16.2
6	.763	.676	.607	-11.4	-20.4
7	.928	.748	.685	-19.4	-26.2
8	1.099	.820	.764	-25.4	-30.5
9	1.343	.892	.842	-33.6	-37.3
10	1.549	.965	.921	-37.7	-40.5
11	1.766	1.037	.999	-41.3	-43.4
12	2.016	1.109	1.078	-45.0	-46.5
13	2.189	1.181	1.156	-46.0	-47.2
14	2.458	1.253	1.235	-49.0	-49.8
15	2.589	1.326	1.313	-48.8	-49.3
16	2.906	1.398	1.392	-51.9	-52.1
17	2.805	1.470	1.470	-47.6	-47.6
18	2.606	1.542	1.549	-40.8	-40.6
19	3.323	1.615	1.627	-51.4	-51.0
20	3.599	1.687	1.706	-53.1	-52.6

a/ Women who were premaritally pregnant or who have used contraception between marriage and last pregnancy are excluded.

b/ Theoretical variance of live births per woman in T years of marriage:

$$A = 0.07224\ T + 0.24210$$
$$B = 0.07848\ T + 0.13590$$

c/ $I = \frac{\text{Theoretical - Observed}}{\text{Observed}} \times 100$

First, the apparent discrepancies between the observed and the theoretical values could be due to deficiencies in the data: for example, still births were not separated from miscarriages, and other errors were introducted due to the fact that the information was collected retrospectively. The retrospective nature of the survey might lead us to suspect some memory bias in reporting all live births by women in the sample during their childbearing period. If so, this would have deflated the observed averages and variances of live births and would have also affected the theoretical series. However, in the present sample, it is believed that the underreporting of live births or infant deaths is negligible because the births reported in the survey were compared with the births registered in the population register maintained by the Taiwanese government. Women for whom the two sources showed different numbers were reinterviewed and the discrepancies were corrected.[18]

A second reason might be the fact that the expressions relating the theoretical average and variance to the duration of marriage assume a constant rate of increase in the average and the variance of live births. Actually this is neither true for the average nor for the variance of live births. Let us consider these points separately.

Due to the increasing frequency of secondary sterility with advancing age, every year some women stop contributing more births, and some women do not produce births as quickly as they did at earlier ages. Thus, the rate of increase in the average number of live births does not remain constant: it starts declining with ascending marriage duration, and finally the rate of increase becomes zero when all women reach the end of their childbearing. In other words, the relationship between the observed number of live births and marriage duration is not linear as predicted by the Perrin and Sheps model, but rather curvilinear. This seems to be the main reason for divergence between the theoretical and observed numbers of live births.

Another consequence of the increasing prevalence of secondary sterility with advancing age is that women married for the same number of years are not homogeneous with respect to the "effective length of reproductive period," which is the period between the onset of marriage and the last live birth prior to the interview. This heterogeneity among women is mainly responsible for the divergence between the observed and the theoretical variances. If we consider women married for at least 15 years then despite their homogeneity with respect to their duration of marriage, they are not homogeneous with respect to their effective reproductive period because this group in-

Figure 2: Theoretical and Observed Variance of Live Births by Duration of Marriage

cludes subgroups of women who had their last live birth during the first, second, third,...fifteenth year after marriage. Thus, women in this group are very different with respect to their attained parity. Now as the length of marriage duration increases, the heterogeneity among women with respect to their attained parity also increases and so does the observed variance of live births. The observed variance will thus continue to increase until all women in the sample reach the end of their reproductive period estimated around 45 years. For the group of women who got married at the age of 20 years, the length of reproductive period will be about 25 years. After 25 years of marriage duration, for this group of women, the rate of increase in the observed variance of live births will be zero, whereas the theoretical variance will still continue to increase with the same constant rate of increase. The needed assumption of an unlimited length for the reproductive period, leads the observed and the theoretical curves to intersect at some point beyond usual observation range after which the theoretical variances will be higher than the observed variances for all years.

A third reason for the apparent divergence between the observed and the theoretical variances of live births might be the violation of the assumption of independence between different states of the model. For obtaining the theoretical variances, all the covariance terms are assumed to be zero; given positive association between different states one might expect positive covariances which would deflate the theoretical variances. The reasons for expecting positive covariances have been discussed elsewhere.[19] Here we will mention them briefly. Due to age dependency, a positive association can be expected between the probability of pregnancy wastage, fecundable periods, and the length of pregnancy intervals following pregnancy losses and following live births. Even with age held constant, a positive association can be expected between the lengths of fecundable and infecundable periods due to their dependences on the outcome of the preceeding pregnancy.

Out of the three reasons discussed above, the first one does not seem to be very important. However, even if one is able to collect perfect data, it seems that the values predicted by the model will not be close to the corresponding observed values because of the nonlinear relationships between marriage duration and the number of live births. The Perrin and Sheps model, in this respect, needs some modifications.

Summary

As an abstract model of fertility process the Perrin and Sheps model necessarily involves simplifying assumptions. In this paper we have compared the values predicted by the model, using parameters estimated from a real population with more variability than the model assumes, with the corresponding observed values. These comparisons show that the theoretical values of averages and variances of live births predicted by the model are not close to the corresponding observed values, even for marriage durations of 10 to 15 years in the middle of the childbearing period, during which the model was supposed to hold. These results suggest that the model's approximate expressions or the corresponding asymptotic expressions giving averages and variances of live births should be used with caution. This may be particularly true when the model is used for estimating the effects of contraception and/or induced abortion on fertility rates, and implications are drawn about the optimum distribution of contraceptive use in a population.

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13. This is a brief version of the detailed description of the Perrin and Sheps model given in their original paper, op. cit.

14. Jain, op. cit.

15. For description of estimation procedure refer to Jain, op. cit.

16. Perrin and Sheps, op. cit., 33.

17. Perrin and Sheps, op. cit., 43-44.

18. For further details see Freedman and Takeshita, op. cit.

19. Jain, op. cit.

Appendix

The two expressions giving the averages and variances of live births are taken from the work of Perrin and Sheps.

The average number of live births during t months of marriage assuming that women start in the fecundable nonpregnant state S_0, is approximately:

$$E\,[LB(t)] \simeq \frac{t}{M(BI)} + \frac{S(BI)}{2[M(BI)]^2} - \frac{M(FBI)}{M(BI)} \qquad (1)$$

and the corresponding variance is approximately:

$$V[LB(t)] \simeq \frac{V(BI)}{[M(BI)]^3}\cdot t + \frac{5[S(BI)]^2}{4[M(BI)]^4} - \frac{2T(BI)}{3[M(BI)]^3} - \frac{S(BI)}{2[M(BI)]^2} - \frac{M(FBI)\,S(BI)}{[M(BI)]^3} + \frac{V(FBI)}{[M(BI)]^2} + \frac{M(FBI)}{M(BI)} \qquad (2)$$

where,

LB Live Birth
BI Completed Birth Interval--Period between two consecutive live births
FBI First Birth Interval--Period between marriage and first live birth
M( ) Mean
V( ) Variance
S( ) Second moment about origin
T( ) Third moment about origin

To evaluate expressions (1) and (2) one needs to estimate the moments of the first birth interval and of completed birth intervals. The simplest way to estimate these moments is to estimate them from the oberved frequency distributions of first birth intervals and completed birth intervals. Another way to estimate these moments is to substitute the estimated values of the parameters of the model in the expressions for estimating the moments of first birth interval and completed birth interval. The procedure for estimating the parameters of the model is described in the author's Ph. D. dissertation. The estimated moments of the first birth interval and the completed birth intervals are shown in Table 3.

Table 3. Estimated Moments of First Birth Interval and Completed Birth Interval in the Absence of Contraception.

Item	Symbol	Estimated Values A	Estimated Values B
Average First Birth Interval	M(FBI)	18.76	18.91
Variance of First Birth Interval	V(FBI)	237.81	173.33
Average Birth Interval	M(BI)	26.67	27.82
Variance of Birth Interval	V(BI)	114.12	140.72
Second Moment of Birth Interval about its Origin	S(BI)	825.28	914.67
Third Moment of Birth Interval about its Orgin	T(BI)	30,991.50	36,279.78

A - Estimated from the observed frequency distributions of FBI and BI.

B - Estimated by substituting the parameters of the model in the expressions of the moments of FBI and BI.